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**Algorithm 1** The SABL algorithm. The feedback distribution  $p(f_t|o_t, a_t, \lambda^*(o_t) = a')$  is described by Equations 1, 2 and 3.  $takeAction(a_t)$  does not return until the episode finishes.

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 $\forall o \in O, a \in A: P[o, a] \leftarrow \frac{1}{|A|}, t \leftarrow 0$ 
while user has not terminated learning do
   $o_t \leftarrow observeWorld()$ 
   $a_t \leftarrow \operatorname{argmax}_{a' \in A} P[o_t, a']$ 
   $takeAction(a_t)$ 
   $f_t \leftarrow lastFeedback()$ 
  for all  $a' \in A$  do
     $P[o_t, a'] \leftarrow p(f_t|o_t, a_t, \lambda^*(o_t) = a')P[o_t, a']$ 
  end for
   $P[o_t, *] \leftarrow normalize(P[o_t, *])$ 
   $t \leftarrow t + 1$ 
end while

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Parameters  $\mu^+$  and  $\mu^-$  encode the trainer's strategy. For example,  $\mu^+=0$  and  $\mu^-=0$  correspond to a balanced strategy where explicit feedback is always given for an action, while  $\mu^+=0$  and  $\mu^-=1$  correspond to a reward-focused strategy, where only actions interpreted as correct receive explicit feedback. Putting these elements together, for episode  $t$ , we have a distribution over the feedback  $f_t$  conditioned on the observation  $o_t$ , action  $a_t$ , and the target policy  $\lambda^*$ ,

$$p(f_t=f^+|o_t, a_t, \lambda^*) = \begin{cases} (1-\epsilon)(1-\mu^+), & \lambda^*(o_t)=a_t \\ \epsilon(1-\mu^+), & \lambda^*(o_t) \neq a_t, \end{cases} \quad (1)$$

$$p(f_t=f^-|o_t, a_t, \lambda^*) = \begin{cases} \epsilon(1-\mu^-), & \lambda^*(o_t)=a_t \\ (1-\epsilon)(1-\mu^-), & \lambda^*(o_t) \neq a_t, \end{cases} \quad (2)$$

$$p(f_t=f^0|o_t, a_t, \lambda^*) = \begin{cases} (1-\epsilon)\mu^+ + \epsilon\mu^-, & \lambda^*(o_t)=a_t \\ \epsilon\mu^+ + (1-\epsilon)\mu^-, & \lambda^*(o_t) \neq a_t. \end{cases} \quad (3)$$

Here,  $f^+$  is an explicit positive feedback,  $f^-$  is an explicit negative feedback, and  $f^0$  is the lack of feedback. Using this model of feedback, SABL computes a maximum likelihood estimate of the trainer's target policy  $\lambda^*$  given the feedback that the user has provided; that is, it computes

$$\operatorname{argmax}_{\lambda} p(h_{1..t}|\lambda^* = \lambda),$$

where  $h_t$  is the training history of actions, observations, and feedback. If a user provides multiple feedbacks during an episode, SABL only considers the most recent, allowing a user to correct a mistaken feedback. Algorithm 1 is an outline of SABL. Note that only the current likelihood distribution is needed to compute the likelihood given a new episode.

### I-SABL: Inferring unknown strategies

While SABL will perform well when it knows the trainer's  $\mu^+$  and  $\mu^-$  parameters, in practice the trainer's strategy will likely be unknown. If, however, the learner knows from explicit feedback the correct action for some observations, it can infer the strategy by looking at the history of feedback for those observations. If, for example, more explicit feedback is given for correct actions than incorrect ones, then

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**Algorithm 2** The I-SABL algorithm. The  $EMupdate(\lambda, h)$  function computes a new policy according to Equation 4.

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 $\lambda \leftarrow randomPolicy(), h \leftarrow \langle \rangle, t \leftarrow 0$ 
while user has not terminated learning do
   $o_t \leftarrow observeWorld()$ 
   $a_t \leftarrow \lambda(o_t)$ 
   $takeAction(a_t)$ 
   $f_t \leftarrow lastFeedback()$ 
   $h \leftarrow \langle h_0, \dots, h_{t-1}, (o, a, f) \rangle$ 
   $\lambda \leftarrow randomPolicy()$ 
  repeat
     $\lambda' \leftarrow \lambda$ 
     $\lambda \leftarrow EMupdate(\lambda, h)$ 
  until  $\lambda = \lambda'$ 
   $t \leftarrow t + 1$ 
end while

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the strategy is likely reward-focused. Under SABL's probabilistic model we can treat the unknown  $\mu$  values representing the trainer's strategy as hidden parameters, and can marginalize over possible strategies to compute the likelihood of a possible target policy  $\lambda$ . Inferring-SABL, or I-SABL, finds a maximum likelihood estimate of the target policy, given the training data. I-SABL attempts to find

$$\operatorname{argmax}_{\lambda} \sum_{s \in S} p(h_{1..t}, s|\lambda^* = \lambda),$$

where  $S$  is the set of possible training strategies ( $\mu^+, \mu^-$  values),  $p(s)$  is uniform for all  $s \in S$ , and  $h_{1..t}$  is the training history up to the current time  $t$ .

The space of possible policies may be exponential in the number of observations, and so algorithms for approximate inference may be needed. Here, we use Expectation Maximization (Dempster, Laird, and Rubin 1977) to compute a maximum likelihood estimate of the target policy, and treat the unknown  $\mu^+$  and  $\mu^-$  parameters as continuous, hidden variables ranging from 0 to 1. The  $i$ th EM update step is then

$$\lambda_{i+1} = \operatorname{argmax}_{\lambda \in P} \int_0^1 \int_0^1 p(\mu^+, \mu^-|h, \lambda_i) \ln p(h, \mu^+, \mu^-|\lambda) d\mu^+ d\mu^-,$$

where  $\lambda_i$  is the current estimate of the policy and  $\lambda_{i+1}$  is the new estimate of the policy. This can be simplified to maximizing the following for a policy's action for each observation  $o$  (details omitted for space):

$$\lambda_{i+1}(o) = \operatorname{argmax}_{a \in A} [\alpha(h_a^{o,+} - h_a^{o,-}) + \beta h_a^{o,0}], \quad (4)$$

where  $h_a^{o,+}$  is the number of positive feedbacks received for observation  $o$  and action  $a$  during training history  $h$ , and  $h_a^{o,-}$  and  $h_a^{o,0}$  are analogous terms for negative feedback and no feedback respectively. Additionally we define

$$\alpha = \ln \left[ \frac{(1-\epsilon)}{\epsilon} \right] \int_0^1 \int_0^1 p(h|\mu^+, \mu^-, \lambda_i) d\mu^+ d\mu^-, \text{ and}$$

$$\beta = \int_0^1 \int_0^1 p(h|\mu^+, \mu^-, \lambda_i) \ln \left[ \frac{(1-\epsilon)\mu^+ + \epsilon\mu^-}{\epsilon\mu^+ + (1-\epsilon)\mu^-} \right] d\mu^+ d\mu^-,$$

values of a simplification of the expectation step, which can be computed once for each EM update. Algorithm 2 is an outline of I-SABL.

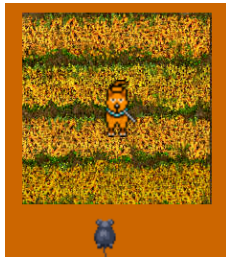


Figure 1: A screenshot of the study interface

## Experiments

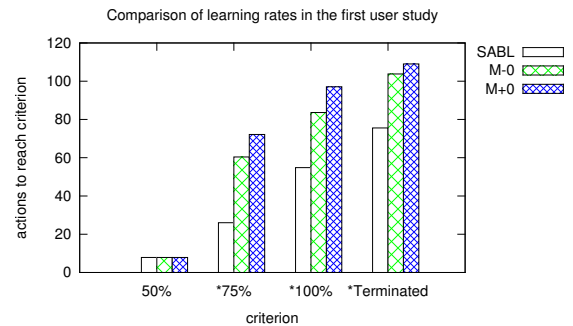
We compare SABL and I-SABL with variants of two algorithms from the literature on learning from human feedback via maximizing numerical reward. Both algorithms maintain an estimate of the expected reward associated with actions for each observation, but differ in their interpretation of no feedback. The first, denoted  $M_{-0}$ , is similar to TAMER (Knox and Stone 2009) and ignores episodes without feedback. The second, denoted  $M_{+0}$ , is similar to COBOT (Isbell et al. 2001) and includes episodes with zero reward — value estimates for actions will return to zero after enough episodes with no feedback. Both algorithms associate  $+1$  with positive and  $-1$  with negative feedback. Unlike SABL and I-SABL,  $M_{-0}$  and  $M_{+0}$  use the cumulative value of all feedback given during an episode.

### User Studies

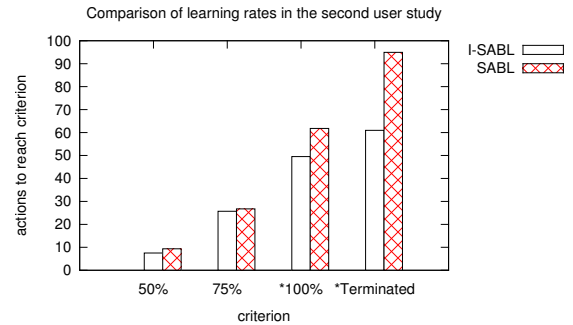
To evaluate their performance when learning from human trainers, we ran an online study in which participants trained learning agents using either SABL (with  $\mu^+ = \mu^- = 0.1$ ), I-SABL,  $M_{-0}$ , or  $M_{+0}$ , to perform a contextual bandit task. We recruited two groups of users via email, online forums, and social networks to participate in our study: university students represented how the average computer-savvy user might train an agent, and amateur dog trainers represented users with experience training using discrete feedback.

Participants were asked to train an animated dog to chase rats away from a corn field. The dog was drawn at the center of the screen (Figure 1), and rats came one at a time every two seconds from three points along each of the four edges (twelve total observations). The learning algorithms were knew nothing about the spatial relationship between observations. The dog (learner) could move up, down, left, or right. Participants were instructed to provide rewards and/or punishments (using the keyboard) to teach the learner to move in the direction the rat was approaching from. Users decided when to terminate experiments, and were told to do so when they felt that either the dog had learned the task sufficiently well, or that it would not be able to learn further.

We ran two studies with this setup, first with participants from both the dog-training forums and the university, then with only participants from dog-training forums. The first study compared SABL against  $M_{-0}$  and  $M_{+0}$ , and had 126 users, of which 71 completed training at least one learner. The second compared I-SABL against SABL and had 43 users, of which 26 completed training at least one learner.



(a) First user study, comparing SABL,  $M_{-0}$  and  $M_{+0}$



(b) Second user study, comparing SABL and I-SABL

Figure 2: Average number of episodes required to teach a policy that was correct for at least 50%, 75%, or 100% of observations, and until the participants terminated the session. (\* indicates that differences were statistically significant for that column)

Our performance measure was the average number of steps it took each agent to reach each of four predetermined criteria. Three of the criteria were when the learner’s estimate of the policy was 50%, 75%, and 100% correct. The fourth criterion was the number of steps before the user terminated the experiment. Results from the first user study show that learners using SABL tended to outperform those using  $M_{-0}$  and  $M_{+0}$ . Figure 2(a) shows the number of steps to reach each of the four criteria. The bars for SABL are lower than their counterparts for the other algorithms, showing that on average the SABL learner took fewer steps to reach the 75%, 100%, and the user termination criteria. Unpaired two sample  $t$ -tests show that the differences between the SABL learner and the  $M_{-0}$  and  $M_{+0}$  learners, for the 75%, 100% and termination criteria, were statistically significant ( $p < 0.05$ ). In addition, a larger percentage of sessions using SABL reached 50%, 75%, and 100% policy correctness than using  $M_{-0}$  or  $M_{+0}$ . Pearson’s  $\chi^2$  tests show that the differences between the number of times the SABL learner and the  $M_{-0}$  and  $M_{+0}$  learners reached the 100% criteria were statistically significant ( $p < 0.01$ ), with the SABL,  $M_{-0}$  and  $M_{+0}$  learners reaching 100% correctness 53%, 17% and 19% of the time respectively.

In the second study, we compared I-SABL against SABL using the same performance criteria to test whether infer-

ring trainers’ strategies improves learning performance. Figure 2(b) shows the number of steps for each algorithm to reach the criteria. Of interest are the very small (statistically insignificant) differences between SABL and I-SABL for the 50% and 75% policy correctness criteria. The difference becomes much larger at the 100% and user-selected termination criteria, where I-SABL reaches each criteria in significantly fewer steps. This is expected, as improvements in learning performance for I-SABL will be most pronounced when the agent has received enough feedback for some observations to infer the trainer’s strategy. Unpaired t-tests show these performance differences are statistically significant, with  $p = 0.01$  for the 100% and  $p < 0.05$  for the termination criteria. A larger percentage of sessions using I-SABL reached 50%, 75%, and 100% policy correctness before termination than using SABL. Pearson’s  $\chi^2$  tests show that the differences between the number of times the I-SABL learner and the SABL learner reached the 100% criteria were significant ( $p < 0.01$ ), with the I-SABL learner reaching 100% policy correctness 50% of the time, and the SABL learner reaching it 23% of the time, respectively.

We note that SABL took more episodes on average to learn in the second study than it did in the first study. We attribute this difference to the fact that users with dog training experience, who were much more common in the second study than in the first, were more likely to use a reward-focused training strategy. As the SABL algorithm assumed a balanced feedback strategy, it ignored episodes without feedback, and so performed more poorly under reward-focused strategies which provided fewer explicit feedbacks.

### Simulated Trainer Experiments

To help understand how strategy inference allows I-SABL to outperform SABL, we ran several experiments with simulated trainers in contextual bandit domains, comparing I-SABL against SABL (with SABL’s  $\mu^+ = \mu^- = 0.1$ ). The simulated trainer chose a target policy at random, and generated feedback using the same probabilistic model underlying SABL and I-SABL. We tested each learning agent on tasks consisting of 2, 5, 10, 15 and 20 observations and 2, 3, or 4 actions. These experiments were conducted for a range of pairs of  $\mu^+$  and  $\mu^-$  values for the simulated trainer. Each  $\mu$  parameter was varied, from 0.0 to 0.8, such that  $\mu^- + \mu^+ \leq 1$ . The trainer’s error rate  $\epsilon = 0.2$ , matching SABL and I-SABL’s assumed value. Learners in these studies took actions at random but kept an estimate of the most likely policy.

The results show that I-SABL is able to take advantage of information from episodes where no explicit feedback is given. Figure 3 shows two curves representing the number of steps it took the SABL and I-SABL agents to find the correct policy, for varying  $\mu$  parameters. The difference in performance between I-SABL and SABL increases (in favor of I-SABL) as the trainer’s  $\mu$  parameters diverge from the balanced strategy that SABL assumes. I-SABL compares well to SABL even when the trainer follows a balanced strategy.

### Sequential Tasks

Results presented so far show SABL and I-SABL in contextual bandit domains. We can also apply these algorithms to

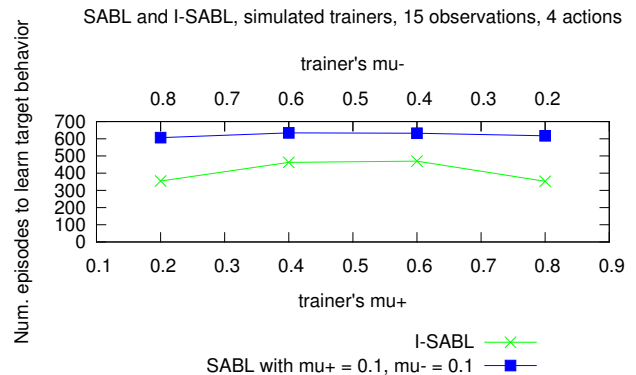


Figure 3: Performance of I-SABL and SABL ( $\mu^- = \mu^+ = 0.1$ ) with simulated trainers. The bottom x-axis is the trainer’s  $\mu^+$ , the top x-axis is  $\mu^-$ , and the y-axis is the number of episodes to find the target policy. As the difference between  $\mu^+$  and  $\mu^-$  grows, so too does the performance difference between SABL and I-SABL.

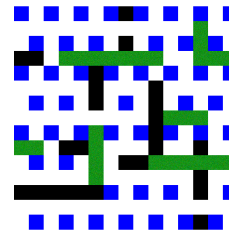


Figure 4: The sequential domain. Blue squares represent possible goal states, black squares represent obstacles of type one and grainy green squares represent obstacles of type two.

sequential decision making domains. For efficiency, we limit the set of policies considered by SABL and I-SABL, by assuming that the trainer teaches an optimal policy for some reward function from a set of reward functions defined over the domain. In a grid world, for example, the trainer could teach the agent to reach a goal location where some reward will be received. Note the use of explicitly defined reward functions here is a syntactic convenience, not a requirement.

We tested SABL and I-SABL for sequential domains in a 15 by 15 grid world with a simulated trainer. The algorithms considered 48 possible goal states, as well as two special kinds of “obstacles”—states the agent could move in to or out of but may have needed to avoid—depending on the reward function. Each possible reward function returned a value of one when the agent reached the goal location,  $-100$  when the it entered an obstacle type that was to be avoided, and zero otherwise. There were four different obstacle conditions (no obstacles, avoid type one, avoid type two, avoid both types), resulting in  $48 \times 4 = 192$  possible reward functions. Figure 4 shows the grid world used. Note that the learners did not actually receive any numeric reward, and so could only learn the correct behavior based on trainer feedback. In the sequential case, SABL and I-SABL simply assumed that the trainer’s target policy was optimal for one of the possible reward functions. Before running SABL and

Table 2: For all algorithm and simulated trainer pairs tested, the average number of steps before the agent correctly identified the intended policy as the most likely, and the average number of explicit feedbacks that were provided before the intended task was identified as the most likely. “N/A” indicates that the algorithm was unable to learn the correct policy in the majority of training runs.

Trainer’s Strategy	Learning Algorithm	Identify Policy	95% Conf. Int.	# Explicit Feedbacks	95% Conf. Interval
balanced	I-SABL	44.4	±11.7	39.1	±10.4
	SABL - balanced feedback	46.7	±9.3	40.5	±8.1
	SABL - reward-focused	67.3	±21.1	60.0	±19.3
	SABL - punishment-focused	65.6	±20.6	58.1	±18.5
reward-focused	I-SABL	68.7	±20.5	54.1	±17.7
	SABL - balanced feedback	152.8	±27.9	71.4	±18.2
	SABL - reward-focused	65	±23.8	50.8	±20.4
	SABL - punishment-focused	N/A	N/A	N/A	N/A
punishment-focused	I-SABL	76.2	±25.4	14.8	±3.9
	SABL - balanced feedback	190.9	±27.3	37.4	±4.5
	SABL - reward-focused	N/A	N/A	N/A	N/A
	SABL - punishment-focused	51.3	±17.9	11.1	±2.8

I-SABL, the reward functions were converted to policies by solving the associated Markov Decision Processes.

In this case SABL and I-SABL only considered a small, finite set of possible  $\mu$  parameter combinations, representing balanced, reward-focused, and punishment-focused trainer strategies. Additionally, to leverage this simplification rather than use EM on the entire feedback history at each step, we adapted I-SABL to update its prior belief in each strategy and policy to the posterior probability distribution given by the most recent feedback and the current distribution over trainer strategies. Trainer strategies were defined by  $\{\mu^+, \mu^-\} = \{0.1, 0.1\}$  for the balanced feedback strategy,  $\{\mu^+, \mu^-\} = \{0.1, 0.9\}$  for the reward-focused strategy, and  $\{\mu^+, \mu^-\} = \{0.9, 0.1\}$  for the punishment-focused strategy. We did not consider the inactive strategy, as it was uncommon in the user study. For all strategies,  $\epsilon = 0.05$ .

Table 2 summarizes the results for all algorithm and trainer strategy pairs. For all simulated trainers, I-SABL and SABL using the correct feedback strategy identified the intended policy the fastest, again demonstrating that I-SABL does not suffer significantly from initial uncertainty about the trainer strategy. When the simulated trainer used a balanced strategy, SABL using incorrect strategy assumptions performed worse, but not significantly worse, likely due the fact that the simulated trainer almost always gave explicit feedback. Regardless of their strategy assumption, SABL learners always interpret explicit feedback in the same way. However, when the trainer does not employ a balanced strategy, incorrect SABL assumptions will be more problematic. If SABL assumes a balanced feedback strategy while the trainer follows a reward-focused strategy, the policy can be learned, but more steps are needed to do so because many steps receive no explicit feedback and so are ignored. If SABL assumes the opposite strategy (e.g., assuming punishment-focused when it is actually reward-focused), then the agent may never learn the correct policy. Assuming the opposite strategy likely performs so poorly because it misinterprets what a lack of feedback means. If SABL assumes a punishment-focused strategy when it’s actually a reward-strategy, it will interpret the lack of feedback when

it’s action is incorrect as evidence that it is correct.

In these results it is interesting to note how few explicit feedbacks are required for I-SABL and SABL (with a correct strategy assumption) to learn when the trainer follows a punishment-focused strategy. As it learns, more of the agent’s actions are correct, resulting in less explicit feedback; since I-SABL (and SABL assuming a punishment-focused strategy) correctly interpret this lack of explicit feedback as positive, it does not hinder learning.

## Conclusion

Initially we argued that most existing work on learning from feedback, which treats trainer feedback as a numerical reward, is not always sufficient for describing the ways in which human trainers provide feedback. We presented empirical data indicating that humans deliver discrete feedback and follow different training strategies when teaching. We have developed two Bayesian learning algorithms, SABL and I-SABL, that can leverage knowledge about those strategies. SABL encodes assumptions about trainer strategies as the probabilities of explicit feedback given the correctness of actions, and I-SABL infers those probabilities online.

Our user studies and simulation experiments demonstrate that the SABL and I-SABL algorithms learn in substantially fewer episodes, and with less feedback, than algorithms modeled after existing numerical-reward-maximizing algorithms. We have demonstrated this advantage even in cases where the trainer’s strategy is initially unknown. Further, we have shown this approach can be effective both in contextual bandit and sequential decision making domains.

Future work would expand the space of feedback strategies considered by SABL and I-SABL to allow temporal delays and variable feedback distributions, and to incorporate knowledge from trainer demonstrations. Future work would also consider how these algorithms can be applied to sequential tasks without enumerating possible reward functions, allowing them to be used in more complex domains.

## Acknowledgements

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